Log Stacks

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1 Introduction

Log geometry was introduced in the late 80s by Fontaine-Illusie, Deligne-Faltings and K. Kato. It offers a more general notion of smoothness, which allows to treat certain objects with mild singularities as if they were smooth. It also provides a functorial way to compactify various moduli spaces: smooth objects often degenerate to logarithmically smooth (but non-smooth) objects over the boundary.
We will start the seminar with some motivation and some background on algebraic stacks. We will then learn about log schemes (schemes equipped with a log structure); after which, we introduce the notion of log structures on stacks over the category of schemes. We will also investigate the relation between log stacks (algebraic stacks with a log structure) and algebraic stacks on the category of log schemes. In particular, we will study the stack of log stable curves $\mathcal{L}\overline{M}_{g,n}$. We finish off the seminar with an important example of log stacks -Artin fans-, which encode the log structure of a log scheme. The seminar is concluded with some research talks on recent results related to log stacks.
Each session will be 45 min + 15 min break + 30-45 min.

2 Prerequisites

Some basic background on categories fibered in groupoids, algebraic spaces and stacks: see last year's seminar on stacks; these notes from Olsson's lectures can also be helpful. No background on log geometry is required.

3 Detailed program

- **Session 1: Overview of the seminar + recall algebraic stacks**.
- **Session 2: Quasi-coherent sheaves on algebraic stacks.** Quasi-coherent sheaves on algebraic spaces ([Ols16, §7.1]), finiteness of cohomology ([Ols16, §7.5]), quasi-coherent sheaves on algebraic stacks ([Ols16, §9]).
- **Session 3: Log schemes 1.** Monoids, log structures, log schemes, morphisms of log schemes, examples: divisorial log structure, toric varieties (in particular $\operatorname{Spec}\mathbb{Z}[P]$ for a monoid P), examples of a non-divisorial log structure (pullback of a log structure, example of a log curve (without definition of a log curve)). See [Tem23, §3].
- **Session 4: Log schemes 2.** Charts (examples from previous talk), definition of fine and saturated log structures, log smoothness/étaleness (through log differentials, with examples), chart criterion. See [Tem23, §3 and §4].
- **Session 5: Log stacks 1.** The stack $\overline{M}_{g,n}$ of stable curves, log curves, table of log curves ([Kat99, §1.8]), going from stacks with log structure to stacks on log schemes and vice versa: the example of $\mathcal{L}\overline{M}_{g,n}$ ([Kat99]), see [Gil12] for the general case.
- **Session 6: Log stacks 2.** Log blowup, idea of the proof that $\mathcal{L}og_X$ is an algebraic stack, local description of $\mathcal{L}og_X$ (toric stacks). See [Ols03].

- **Session 7: Log stacks 3 (Artin fans).** The category of Artin fans, the Artin fan of a logarithmic scheme, Artin fans and functoriality. See [ACMW17, §3], [ACM$^+$16, §5] and [AW18].
- **Session 8: Research talks.** TBD.

References

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[ACMW17] Dan Abramovich, Qile Chen, Steffen Marcus, and Jonathan Wise. Boundedness of the space of stable logarithmic maps. *Journal of the European Mathematical Society*, 19(9):2783–2809, 2017.

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