1. The Étale fundamental group
(Supervised by Sara Mehidi – Block 2 only)

Given an algebraic variety, one may naturally seek an analogue of a topological covering space. In algebraic geometry, the concept of an étale morphism, and specifically an étale cover, provides the right framework for this analogy. Just as covering maps in topology can be thought of as local homeomorphisms with certain additional conditions, étale morphisms behave like local isomorphisms on varieties.

In this setting, the étale fundamental group plays a role similar to the topological fundamental group. For a connected algebraic variety, the étale fundamental group classifies finite étale covers in the same way that the topological fundamental group classifies covering spaces of a topological space.

The aim of this project is to explore these ideas, work through key examples to illustrate the analogies with the topological setting, and investigate applications.

References:

- J.S. Milne; Lectures on Étale Cohomology (§1.2 and §1.3).
- Master thesis in the University of Bordeaux: Galois categories and étale covers.

Algebraic geometry